MMO 2020

MetriX Mathematical Olympiad

DAY 2

- The rules are same as the rules of the Actual IMO contest. Each problem is containing 7 points. There are partial markings so be sure that you solution is well explained also if your progress is right up to some part you will get some partial marks.

- It is advised to take the mock in the 4.5 hours as the standard time of IMO, however the submission deadline is two weeks. You can make a pdf of your latex solution or you can submit handwritten solutions but be sure its neat and clean subjective solutions.

- The problems are mostly original so search function won't help you much, if you find any problem which is known, please keep it with you or just PM us.

- This problems will be posted on HSO(High school Olympiad Forum) after the exam is over, just use the tag MMO and you can find it on HSO later.

- Please avoid any unfair means, try the problems yourselves. Also note that you can send the solutions one by one but it must be in the same pm and add all members - Aritra12, TLP.39, MNJ2357, Functional_Equation, Amar_04 and Mr.C.

MMO 2020

1st MetriX Mathematical Olympiad

Day 2(English)

Thursday, 5th November 2020

MMO4. A permutation of $[n]$ is a list of numbers from 1 to n written in a row. A permutation is said to be jumpy if no two adjacent elements differ by exactly 1. A jumpy permutation π is said to be boring if the following holds: There exists an element i in π such that if we remove i from π and re-number the remaining numbers in the row, in ascending order of their value, from 1 to $n-1$, then this permutation on $n-1$ elements is jumpy. For example, 31524 is jumpy but is boring because we can remove 1 to get 3524 and can renumber this as 2413 (because $2 < 3 < 4 < 5$), which is jumpy. Note that we could also have shown that 31524 is boring by removing the element 4 and re-numbering 3152 as 3142.The permutation on 0 elements and the permutation on 1 element are both taken as jumpy.Find the number of jumpy permutations on n elements that are not boring.

MMO5. ABC be a scalene triangle. Let D be a point on $\overline{BC}$ such that $\overline{AD}$ is the angle bisector of $\angle BAC$. Let M be the midpoint of $\widehat{BAC}$ of the circumcircle of $\odot(ABC)$. Let $\overline{DN} \perp \overline{BC}$ where N lies on $\overline{AM}$. K be a point on segment NM such that $3KM = KN$. Let P, Q be two points on $\overline{AM}$ such that $KP = KQ$ and $BCPQ$ lie on a circle ω. Show that the reflection of D over A lies on ω.

MMO6. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$f(x^2 + 2f(xy)) = xf(x+y) + f(xy)$$

for all $x, y \in R^+$

Language : English

Time: 4 hours and 30 minutes
Each problem is worth 7 points